Quantifying Skype User Satisfaction

Kuan-Ta Chen¹², Chun-Ying Huang¹, Polly Huang¹³, and Chin-Laung Lei¹³

¹Department of Electrical Engineering, National Taiwan University ²Institute of Information Science, Academia Sinica ³Graduate Institute of Networking and Multimedia, National Taiwan University

ABSTRACT

The success of Skype has inspired a generation of peer-topeer-based solutions for satisfactory real-time multimedia services over the Internet. However, fundamental questions, such as whether VoIP services like Skype are good enough in terms of user satisfaction, have not been formally addressed. One of the major challenges lies in the lack of an easily accessible and objective index to quantify the degree of user satisfaction.

In this work, we propose a model, geared to Skype, but generalizable to other VoIP services, to quantify VoIP user satisfaction based on a rigorous analysis of the call duration from actual Skype traces. The User Satisfaction Index (USI) derived from the model is unique in that 1) it is composed by objective source- and network-level metrics, such as the bit rate, bit rate jitter, and round-trip time, 2) unlike speech quality measures based on voice signals, such as the PESQ model standardized by ITU-T, the metrics are easily accessible and computable for real-time adaptation, and 3) the model development only requires network measurements, i.e., no user surveys or voice signals are necessary. Our model is validated by an independent set of metrics that quantifies the degree of user interaction from the actual traces.

Categories and Subject Descriptors

H.4.3 [Information Systems Applications]: Communications Applications—*Computer conferencing, teleconferencing, and videoconferencing*; G.3 [Numerical Analysis]: Probability and Statistics—*Survival Analysis*; H.1.2 [Models and Principles]: User/Machine Systems—*Human factors*

General Terms

Human Factors, Measurement, Performance

Keywords

Human Perception, Internet Measurement, Survival Analysis, Quality of Service, VoIP, Wavelet Denoising

1. INTRODUCTION

There are over 200 million Skype downloads and approximately 85 million users worldwide. The user base is growing at more than 100,000 a day, and there are 3.5 to 4 million active users at any one time¹². The phenomenal growth of Skype has not only inspired a generation of applicationlevel solutions for satisfactory real-time multimedia services over the Internet, but also stunned the market observers worldwide with the recent US\$ 4.1 billion deal with eBay³. Network engineers and market observers study Skype for different reasons. The former seek ways to enhance user satisfaction, while the latter collect information to refine their predictions of the growth of the user base. They both, however, need the answer to a fundamental question: Is Skype providing a good enough voice phone service to the users, or is there still room for improvement?

To date, there has not been a formal study that quantifies the level of user satisfaction with the Skype voice phone service. The difficulties lie in 1) the peer-to-peer nature of Skype, which makes it difficult to capture a substantial amount of traffic for analysis; and 2) existing approaches to studying user satisfaction rely on speech-signal-level information that is not available to parties other than the call participants. Furthermore, studies that evaluate the perceptual quality of audio are mostly signal-distortion-based [9– 11]. This approach has two drawbacks: 1) it usually requires access to signals from both ends, the original and the degraded signals, which is not practical in VoIP applications; 2) it cannot take account of factors other than speech signal degradation, e.g., variable listening levels, sidetone/talk echo, and conversational delay.

We propose an objective, perceptual index for measuring Skype user satisfaction. The model, called the User Satisfaction Index (USI), is based on a rigorous analysis of the call duration and source- and network-level QoS metrics. The specific model presented in this paper is geared to Skype,

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¹http://www.voipplanet.com/solutions/article.php/3580131
²http://www.skypejournal.com/blog/archives/2005/05/
3_million_skype_1.php

³http://gigaom.com/2005/09/11/skype-ebay-happening/

but the methodology is generalizable to other VoIP and interactive real-time multimedia services. Our model is unique in that the parameters used to construct the index are easy to access. The required information can be obtained by passive measurement and ping-like probing. The parameters are also easy to compute, as only first- and second-moment statistics of the packet counting process are needed. There is no need to consider voice signals. Therefore, user satisfaction can be assessed online. This will enable any QoSsensitive application to adapt in real time its source rate, data path, or relay node for optimal user satisfaction.

To validate the index, we compare the proposed USI with an independent set of metrics that quantify the degree of voice interactivity from actual Skype sessions. The basic assumption is that the more smoothly users interact, the more satisfied they will be. The level of user interaction is defined by the responsiveness, response delay, and talk burst length. Speech activity is estimated by a wavelet-based algorithm [6] from packet size processes. The strong correlation observed between the interactivity of user conversations and USI supports the representativeness of the USI.

By deriving the objective, perceptual index, we are able to quantify the relative impact of the bit rate, the compound of delay jitter and packet loss, and network latency on Skype call duration. The importance of these three factors is approximately 46%:53%:1% respectively. The delay jitter and loss rate are known to be critical to the perception of realtime applications. To our surprise, network latency has relatively little effect, but the source rate is almost as critical as the compound of the delay jitter and packet loss. We believe these discoveries indicate that adaptations for a stable, higher bandwidth channel are likely the most effective way to increase user satisfaction in Skype. The selection of relay nodes based on network delay optimization, a technique often used to find a quality detour by peer-to-peer overlay multimedia applications, is less likely to make a significant difference for Skype in terms of user satisfaction.

Our contribution is three-fold: 1) We devise an objective and perceptual user satisfaction index in which the parameters are all easily measurable and computable online; 2) we validate the index with an independent set of metrics for voice interactivity derived from user conversation patterns; and 3) we quantify the influence of the bit rate, jitter and loss, and delay on call duration, which provides hints about the priority of the metrics to tune for optimal user satisfaction with Skype.

The remainder of this paper is organized as follows. Section 2 describes related works. We discuss the measurement methodology and summarize our traces in Section 3. In Section 4, we derive the USI by analyzing Skype VoIP sessions, especially the relationship between call duration and source-/network-level conditions. In Section 5, we validate the USI with an independent set metrics based on speech interactivity. Finally, Section 6 draws our conclusion.

2. RELATED WORK

Objective methods for assessing speech quality can be classified into two types: referenced and unreferenced. Referenced methods [9,11] measure distortion between original and degraded speech signals and map the distortion values to mean opinion scores (MOS). However, there are two problems with such model: 1) both the original and the degraded signals must be available, and 2) it is difficult to synchro-

Table 1:	Comparison	of the	proposed	\mathbf{USI}	\mathbf{and}	\mathbf{the}
objective	measures of	speech	quality			

	USI	speech quality measures			
to quantify	user satisfaction	speech quality			
built upon [†]	call duration	subjective MOS			
predictors	QoS factors	distortion of signals			
the near ange workship used in the model development					

⁺ the response variable used in the model development

nize the two signals. Unreferenced models [10], on the other hand, do not have the above problems, as only the degraded signal is required. The unreferenced models, however, do not capture human perception as well as the referenced models.

The USI model and the measures of speech quality although aim similarly at providing objective metrics to quantify user perception, however, they have a number of substantial differences: 1) the USI model is based on call duration, rather than speech quality; therefore, factors other than speech quality, such as listening volume and conversational delay [11], can also be captured by USI; and 2) rather than relying on subjective surveys, the USI model is based on passive measurement, so it can capture *subconscious reactions* that listeners are even unaware of. Table 1 summarizes the major differences.

3. TRACE COLLECTION

In this section, we describe the collection of Skype VoIP sessions and their network parameters. We first present the network setup and filtering method used in the traffic capture stage. The algorithm for extracting VoIP sessions from packet traces is then introduced, followed by the strategy to sample path characteristics. Finally, we summarize the collected VoIP sessions.

3.1 Network Setup

Because of the peer-to-peer nature of Skype, no one network node can see traffic between any two Skype hosts in the world. However, it is still possible to gather Skype traffic related to a particular site. To do so, we set up a packet sniffer that monitors all traffic entering and leaving a campus network, as shown in Fig. 1. The sniffer is a FreeBSD 5.2 machine equipped with dual Intel Xeon 3.2G processors and one gigabyte memory. As noted in [12], two Skype nodes can communicate via a relay node if they have difficulties establishing sessions. Also, a powerful Skype node is likely to be used as the relay node of VoIP sessions if it has been up for a sufficient length of time. Therefore we also set up a powerful Linux machine to elicit more relay traffic during the course of trace collection.

3.2 Capturing Skype Traffic

Given the huge amount of monitored traffic and the low proportion of Skype traffic, we use two-phase filtering to identify Skype VoIP sessions. In the first stage, we filter and store possible Skype traffic on the disk. Then in the second stage, we apply an off-line identification algorithm on the captured packet traces to extract actual Skype sessions.

To detect possible Skype traffic in real time, we leverage some known properties of Skype clients [1, 12]. First of all, Skype does not use any well-known port number, which is



Figure 1: The network setup for VoIP session collection

one of the difficulties in distinguishing Skype traffic from that of other applications. Instead, it uses a dynamic port number in most communications, which we call the "Skype port" hereafter. Skype uses this port to send all outgoing UDP packets and accept incoming TCP connections and UDP packets. The port is chosen randomly when the application is installed and can be configured by users. Secondly, in the login process, Skype submits HTTP requests to a well-known server, ui.skype.com. If the login is successful, Skype contacts one or more *super nodes* listed in its host cache by sending UDP packets through its Skype port.

Based on the above knowledge, we use a heuristic to detect Skype hosts and their Skype ports. The heuristic works as follows. For each HTTP request sent to ui.skype.com, we treat the sender as a Skype host and guess its Skype port by inspecting the port numbers of outgoing UDP packets sent within the next 10 seconds. The port number used most frequently is chosen as the Skype port of that host. Once a Skype host has been identified, all peers that have bi-directional communication with the Skype port on that the host are also classified as Skype hosts. With such a heuristic, we maintained a table of identified Skype hosts and their respective Skype ports, and recorded all traffic sent from or to these (host, port) pairs. The heuristic is not perfect because it also records non-voice-packets and may collect traffic from other applications by mistake. Despite the occasional false positives, it does reduce the number of packets we need to capture to only 1-2% of the number of observed packets in our environment. As such, it is a simple filtering method that effectively filters out most unwanted traffic and reduces the overhead of off-line processing.

3.3 Identification of VoIP Sessions

Having captured packet traces containing possible Skype traffic, we proceed to extract true VoIP sessions. We define a "flow" as a succession of packets with the same five-tuple (source and destination IP address, source and destination port numbers, and protocol number). We determine whether a flow is active or not by its moving average packet rate. A flow is deemed active if its rate is higher than a threshold 15 pkt/sec, and considered inactive otherwise. The average packet rate is computed using an exponential weighted moving average (EWMA) as follows:

$$A_{i+1} = (1 - \alpha)A_i + \alpha I_i,$$

where I_i represents the average packet rate of the *i*-th second

of the flow and A_i is the average rate. The weight α is set at 0.15 when the flow is active and 0.75 when the flow is inactive. The different weights used in different states allow the start of a flow to be detected more quickly [12].

An active flow is regarded as a valid VoIP session if all the following criteria are met:

- The flow's duration is longer than 10 seconds.
- The average packet rate is within a reasonable range, (10, 100) pkt/sec.
- The average packet size is within (30, 300) bytes. Also, the EWMA of the packet size process (with $\alpha = 0.15$) must be within (35, 500) bytes all the time.

After VoIP sessions have been identified, each pair of sessions is checked to see if it can form a *relayed session*, i.e., these two flows are used to convey the same set of VoIP packets with the relay node in our campus network. We merge a pair of flows into a relayed session if the following conditions are met: 1) the flows' start and finish time are close to each other with errors less than 30 seconds; 2) the ratio of their average packet rates is smaller than 1.5; and 3) their packet arrival processes are positively correlated with a coefficient higher than 0.5.

3.4 Measurement of Path Characteristics

As voice packets may experience delay or loss while transmitting over the network, the path characteristics would undoubtedly affect speech quality. However, we cannot deduce round-trip times (RTT) and their jitters simply from packet traces because Skype voice packets are encrypted and most of them are conveyed by UDP. Therefore, We send out probe packets to measure paths' round-trip times while capturing Skype traffic. In order to minimize the possible disturbance caused by active measurement, probes are sent in batches of 20 with exponential intervals of mean 1 second. Probe batches are sent at two-minute intervals for each active flow. While "ping" tasks are usually achieved by ICMP packets, many routers nowadays discard such packets to reduce load and prevent attacks. Fortunately, we find that, certain Skype hosts respond to traceroute probes sent to their Skype ports. Thus, to increase the yield rate of RTT samples, traceroute-like probes, which are based on UDP, are also used in addition to ICMP probes.

3.5 Trace Summary

The trace collection took place over two months in late 2005. We obtained 634 VoIP sessions, of which 462 sessions were usable as they had more than five RTT samples. Of the 462 sessions, 253 were directly-established and 209 were relayed. A summary of the collected sessions is listed in Table 2. One can see from the table that median of the relayed session durations is significantly shorter than that of the direct sessions, as their 95% confidence bands do not overlap. We believe the discrepancy could be explained by various factors, such as larger RTTs or lower bit rates. The relationship between these factors and the session/call duration is investigated in detail in the next section.

4. ANALYSIS OF CALL DURATION

In this section, based on a statistical analysis, we posit that call duration is significantly correlated with QoS factors, including the bit rate, network latency, network delay

Table 2: Summary of collected VoIP sessions

Category	Calls	$Hosts^{\dagger}$	Cens.	TCP	$Duration^{\ddagger}$	Bit Rate (mean/std)	Avg. RTT (mean/std)
Direct	253	240	1	7.1%	$(6.43, 10.42) \min$	32.21 Kbps / 15.67 Kbps	157.3 ms / 269.0 ms
Relayed	209	369	5	9.1%	$(3.12, 5.58) \min$	29.22 Kbps / 10.28 Kbps	376.7 ms / 292.1 ms
Total	462	570	6	8.0%	$(5.17, 7.70) \min$	30.86 Kbps / 13.57 Kbps	256.5 ms / 300.0 ms
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[†] Number of involved Skype hosts in VoIP sessions, including relay nodes used (if any).

 ‡ The 95% confidence band of median call duration.

variations, and packet loss. We then develop a model to describe the relationship between call duration and QoS factors. Assuming that call duration implies the conversation quality users perceive, we propose an objective index, the User Satisfaction Index (USI) to quantify the level of user satisfaction. Later, in Section 5, we will validate the USI by voice interactivity measures inferred from user conservations, where both measures strongly support each other.

4.1 Survival Analysis

In our trace (shown in Table 2), 6 out of 462 calls were censored, i.e., only a portion of the calls was observed by our monitor. This was due to accidental disk or network outage during the trace period. Censored observations should be also used because longer sessions are more likely to be censored than shorter session. Simply disregarding them will lead to underestimation. Additionally, while regression analysis is a powerful technique for investigating relationships among variables, the most commonly used linear regression is not appropriate for modeling call duration because the assumption of normal errors with equal variance is violated. However, with proper transformation, the relationships of session time and predictors can be described well by the Cox Proportional Hazards model [3] in survival analysis. For the sake of censoring and the Cox regression model, we adopt methodologies as well as terminology in survival analysis in this paper.

4.2 Effect of Source Rate

Skype uses a wideband codec that adapts to the network environment by adjusting the bandwidth used. It is generally believed that Skype uses the iSAC codec provided by Global IP Sound. According to the white paper of iSAC⁴, it automatically adjusts the transmission rate from a low of 10 Kbps to a high of 32 Kbps. However, most of the sessions in our traces used 20–64 Kbps. A higher source rate means that more quality sound samples are sent at shorter intervals so that the receiver gets better voice quality. Therefore, we expect that users' conversation time will be affected, to some extent, by the source rate chosen by Skype.

Skype adjusts the voice quality by two orthogonal dimensions: the frame size (30–60 ms according to the iSAC white paper), and the encoding bit rate. The frame size directly decides the sending rate of voice packets; for example, 300 out of 462 sessions use a frame size of 30 ms in both directions, which corresponds to about 33 packets per second. Because packets may be delayed or dropped in the network, we do not have exact information about the source rate of remote Skype hosts, i.e., nodes outside the monitored network. Assuming the loss rate is within a reasonable range,



Figure 2: Survival curves for sessions with different bit rate levels

say less than 5%, we use the received data rate as an approximation of the source rate. We find that packet rates and bit rates are highly correlated (with a correlation coefficient ≈ 0.82); however, they are not perfectly proportional because packet sizes vary. To be concise, in the following, we only discuss the effect of the bit rate because 1) it has a higher correlation with call duration; and, 2) although not shown, the packet rate has a similar (positive) correlation with session time.

We begin with a fundamental question: "Does call duration differ significantly with different bit rates?" To answer this question, we use the estimated survival functions for sessions with different bit rates. In Fig. 2, the survival curves of three session groups, divided by 25 Kbps (15%) and 35 Kbps (60%), are plotted. The median session time of groups 1 and 3 are 2 minutes and 20 minutes, respectively, which gives a high ratio of 10. We can highlight this difference in another way: while 30% of calls with bit rates > 35 Kbps last for more than 40 minutes, only 3% of calls hold for the same duration with low bit rates (< 25 Kbps).

The Mantel-Haenszel test (also known as the log-rank test) [8] is commonly used to judge whether a number of survival functions are statistically equivalent. The log-rank test, with the null hypothesis that all the survival functions are equivalent, reports $p = 1 - \Pr_{\chi^2,2}(58) \approx 2.7e - 13$, which strongly suggests that call duration varies with different levels of bit rate. To reveal the relationship between the bit rate and call duration in more detail, the median time and their standard errors of sessions with increasing bit rates are plotted in Fig. 3. The trend of median duration shows a strong, consistent, positive, correlation with the bit rate. Before concluding that the bit rate has a pronounced effect

⁴http://www.globalipsound.com/datasheets/iSAC.pdf



Figure 3: Correlation of bit rate with session time

on call duration, however, we remark that the same result could also be achieved if Skype always chooses a low bit rate initially, and increases it gradually. The hypothesis is not proven because a significant proportion of sessions (7%) are of short duration (< 5 minutes) and have a high bit rate (> 40 Kbps).

4.3 Effect of Network Conditions

In addition to the source rate, network conditions are also considered to be one of the primary factors that affect voice quality. In order not to disturb the conversation of the Skype users, the RTT probes (cf. Section 3.4) were sent at 1 Hz, a frequency that is too low to capture delay jitters due to queueing and packet loss. So we must seek some other metric to grade the interference of the network. Given that 1) Skype generates VoIP packets regularly, and 2) the frequency of VoIP packets is relatively high, so the fluctuations in the data rate observed at the receiver should reflect network delay variations to some extent. Therefore, we use the standard deviation of the bit rate sampled every second to represent the degree of network delay jitters and packet loss. For brevity, we use jitter to denote the standard deviation of the bit rate, and packet rate jitter, or *pr.jitter*, to denote the standard deviation of the packet rate.

4.3.1 Effect of Round-Trip Times

We divide sessions into three equal-sized groups based on their RTTs, and compare their lifetime patterns with the estimated survival functions. As a result, the three groups differ significantly (p = 3.9e - 6). The median duration of sessions with RTTs > 270 ms is 4 minutes, while sessions with RTTs between 80 ms and 270 ms and sessions with RTTs < 80 ms have median duration of 5.2 and 11 minutes, respectively.

4.3.2 Effect of Jitter

We find that variable *jitter*, which captures the level of network delay variations and packet loss, has a much higher correlation with call duration than round-trip times. As shown in Fig. 4, the three session groups, which are divided by jitters of 1 Kbps and 2 Kbps, have median time of 3, 11, and 21 minutes, respectively. The p-value of the equivalence test of these groups is $1 - \Pr_{\chi^2,2}(154) \approx 0$. The correlation plot, depicted in Fig. 5, shows a consistent and significant



Figure 4: Survival curves for sessions with different levels of jitter



Figure 5: Correlation of jitter with session time

downward trend in jitter versus time. Although not very pronounced, the jitter seems to have a "threshold" effect when it is of low magnitude. That is, a negative correlation between jitter and session time is only apparent when the former is higher than 0.5 pkt/sec. Such threshold effects, often seen in factors that capture human behavior, are plausible because listeners may not be aware of a small amount of degradation in voice quality.

4.4 Regression Modeling

We have shown that most of the QoS factors we defined, including the source rate, RTT, and jitter, are related to call duration. However, we note that correlation analysis does not reveal the true impact of individual factors because of *the collinearity of factors*. For example, given that the bit rate and jitter are significantly correlated (with $p \approx 2e - 6$), if both factors are related to call duration, which one is the true source of user dissatisfaction is unclear. Users could be particularly unhappy because of one of the factors, or be sensitive to both of them.

To separate the impact of individual factors, we adopt regression analysis to model call duration as the response to QoS factors. Given that linear regression is inappropriate for modeling call duration, we show that the Cox model provides a statistically sound fit for the collected Skype ses-

	br	pr	jitter	pr.jitter	pktsize	rtt
br	*	+++	++		+++	—
pr	+++	*	_		+	
jitter	++	_	*	+++	+++	
pr.jitter			+++	*		
pktsize	+++	+	+++		*	
rtt	_					*

Table 3: The directions and levels of correlation be-tween pairs of QoS factors

 † +/-: positive or negative correlation.

[‡] Symbol #: p-value is less than 5e-2, 1e-3, 1e-4, respectively.

sions. Following the development of the model, we propose an index to quantify user satisfaction and then validate the index by prediction.

4.4.1 The Cox Model

The Cox proportional hazards model [3] has long been the most used procedure for modeling the relationship between factors and *censored* outcomes. In the Cox model, we treat QoS factors, e.g., the bit rate, as risk factors or covariates; in other words, as variables that can cause failures. In this model, the hazard function of each session is *decided completely by a baseline hazard function and the risk factors related to that session*. We define the risk factors of a session as a risk vector \mathbf{Z} . The regression equation is defined as

$$h(t|\mathbf{Z}) = h_0(t) \exp(\beta^t \mathbf{Z}) = h_0(t) \exp(\sum_{k=1}^p \beta_k Z_k),$$
 (1)

where $h(t|\mathbf{Z})$ is the hazard rate at time t for a session with risk vector \mathbf{Z} ; $h_0(t)$ is the baseline hazard function computed during the regression process; and $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)^t$ is the coefficient vector that corresponds to the impact of risk factors. Dividing both sides of Equation 1 by $h_0(t)$ and taking the logarithm, we obtain

$$\log \frac{h(t|\mathbf{Z})}{h_0(t)} = \beta_1 Z_1 + \dots + \beta_k Z_k = \sum_{k=1}^p \beta_k Z_k = \beta^t \mathbf{Z}, \quad (2)$$

where Z_p is the *p*th factor of the session. The right side of Equation 2 is a linear combination of covariates with weights set to the respective regression coefficients, i.e., it is transformed into a *linear regression equation*. The Cox model possesses the property that, if we look at two sessions with risk vectors **Z** and **Z'**, the hazard ratio (ratio of their hazard rates) is

$$\frac{h(t|\mathbf{Z})}{h(t|\mathbf{Z}')} = \frac{h_0(t)\exp[\sum_{k=1}^p \beta_k Z_k]}{h_0(t)\exp[\sum_{k=1}^p \beta_k Z'_k]}$$
$$= \exp[\sum_{k=1}^p \beta_k (Z_k - Z'_k)], \quad (3)$$

which is a time-independent constant, i.e., the hazard ratio of the two sessions is independent of time. For this reason the Cox model is often called the *proportional hazards model*. On the other hand, Equation 3 imposes the strictest conditions when applying the Cox model, because the validity of the model relies on the assumption that the *hazard rates for any two sessions must be in proportion all the time*.

4.4.2 Collinearity among Factors

Although we can simply put all potential QoS factors into a regression model, the result would be ambiguous if the predictors were strongly interrelated [7]. Now that we have seven factors, namely, the bit rate (br), packet rate (pr), *jitter*, *pr.jitter*, packet size (pktsize), and round-trip times (rtt), we explain why not all of them can be included in the model simultaneously.

Table 3 provides the directions and levels of interrelation between each pair of factors, where the p-value is computed by Kendall's τ statistic as the pairs are not necessarily derived from a bivariate normal distribution. However, Pearson's product moment statistic yields similar results. We find that 1) the bit rate, packet rate, and packet size are strongly interrelated; and 2) jitter and packet rate jitter are strongly interrelated. By comparing the regression coefficients when correlated variables are added or deleted, we find that the interrelation among QoS factors is very strong so that variables in the same collinear group could interfere with each other. To obtain an interpretable model, only one variable in each collinear group can remain. As a result, only the bit rate, jitter, and RTT are retained in the model, as the first two are the most significant predictors compared with their interrelated variables.

4.4.3 Sampling of QoS Factors

In the regression modeling, we use *a scalar value* for each risk factor to capture user perceived quality. QoS factors, such as the round-trip delay time, however, are usually not constant, but vary during a call. To extract a representative value for each factor in a session, which is resemble to feature vector extraction in pattern recognition, will be a key to how well the model can describe the observed sessions.

Intuitively, the values averaged across the whole session time would be a good choice. However, extreme conditions may have much more influence on user behavior than ordinary conditions. For example, users may hang up a call earlier because of serious network lags in a short period, but be insensitive to mild and moderate lags that occur all the time. To derive the most representative risk vector, we propose three measures to account for network quality, namely, the minimum, the average, and the maximum, by two-level sampling. That is, the original series s is first divided into sub-series of length w, from which network conditions are sampled. This sub-series approach confines measure of network quality within time spans of w, thus excludes the effect of large-scale variations. The minimum, average, and maximum measures are then taken from sampled QoS factors which has length $\lceil |s|/w \rceil$. One of the three measures will be chosen depending on their ability to describe the user perceived experience during a call.

We evaluate all kinds of measures and window sizes by fitting the extracted QoS factors into the Cox model and comparing the model's log-likelihood, i.e., an indicator of goodness-of-fit. Finally, the maximum bit rate and minimum jitter are chosen, both sampled with a window of 30 seconds. The short time window implies that users are more sensitive to short-term, rather than long-term, behavior of network quality, as the latter may have no influence on voice quality at all. The sampling of the bit rate, jitter, and RTT consistently chooses the value that represents the best quality a user experienced. This interesting finding may be further verified by cognitive models that could determine whether



Figure 6: The functional form of the bit rate factor

the best or the worst experience has a more dominant effect on user behavior.

4.4.4 Model Fitting

For a continuous variable, the Cox model assumes a linear relationship between the covariates and the hazard function, i.e., it implies that the ratio of risks between a 20 Kbps- and a 30 Kbps-bit rate session is the same as that between a 40 Kbps- and 50 Kbps-bit rate session. Thus, to proceed with the Cox model, we must ensure that our predictors have a linear influence on the hazard functions.

We investigate the impact of the covariates on the hazard functions with the following equation:

$$E[s_i] = \exp(\boldsymbol{\beta}^t f(\mathbf{Z})) \int_0^\infty I(t_i \ge s) h_0(s) ds, \qquad (4)$$

where s_i is the censoring status of session *i*, and f(z) is the estimated functional form of the covariate *z*. This corresponds to a Poisson regression model if $h_0(s)$ is known, where the value of $h_0(s)$ can be approximated by simply fitting a Cox model with unadjusted covariates. We can then fit the Poisson model with smoothing spline terms for each covariate [13]. If the covariate has a linear impact on the hazard functions, the smoothed terms will approximate a straight line.

In Fig. 6(a), we plot the fitted splines, as well as their two-standard-error confidence bands, for the bit rate factor. From the graph, we observe that the influence of the bit rate is not proportional to its magnitude (note the change of slope around 35 Kbps); thus, modeling this factor as linear would not provide a good fit. A solution for nonproportional variables is *scale transformation*. As shown in Fig. 6(b), the logarithmic variable, *br.log*, has a smoother and approximately proportional influence on the failure rate. This indicates that the failure rate is proportional to the *scale of the bit rate*, rather than its magnitude. A similar



Figure 7: The functional form of the jitter factor

Table 4: Coefficients in the final model

Variable	Coef	e^{Coef}	Std. Err.	z	P > z
br.log	-2.15	0.12	0.13	-16.31	0.00e+00
jitter.log	1.55	4.7	0.09	16.43	0.00e+00
rtt	0.36	1.4	0.18	2.02	4.29e-02

situation occurs with the *jitter* factor, i.e., the factor also has a non-linear impact, but its impact is approximately linear by taking logarithms, as shown in Fig. 7. On the other hand, the RTT factor has an approximate linear impact so that there is no need to adjust for it.

We employ a more generalized Cox model that allows time-dependent coefficients [13] to check the proportional hazard assumption by hypothesis tests. After adjustment, none of covariates reject the linearity hypothesis at significance level 0.1, i.e., the transformed variables have an approximate linear impact on the hazard functions. In addition, we use the Cox and Snell residuals r_i (for session *i*) to assess the overall goodness-of-fit of the model [4]. We find that, except for a few sessions that have unusual call duration, most sessions fit the model very well; therefore, the adequacy of the fitted model is confirmed.

4.4.5 Model Interpretation

The regression coefficients, β , along with their standard errors and significance values of the final model are listed in Table 4. Contrasting them with Fig. 6 and Fig. 7 reveals that the coefficients $\beta_{br.log}$ and $\beta_{jitter.log}$ are simply the slopes in the linear regression of the covariate versus the hazard function. β can be physically interpreted by the hazard ratios (Equation 3). For example, assuming two Skype users call their friends at the same time with similar bit rates and round-trip times to the receivers, but



Figure 8: Relative influence of different QoS factors for each session

the jitters they experience are 1 Kbps and 2 Kbps respectively, the hazard ratio of the two calls can be computed by $\exp((log(2) - log(1)) \times \beta_{jitter.log}) \approx 2.9$. That is, as long as both users are still talking, in every instant, the probability that user 2 will hang up is 2.9 times the probability that user 1 will do so.

The model can also be used to quantify the relative influence of QoS factors. Knowing which factors have more impact than others is beneficial, as it helps assign resources appropriately to derive the maximum marginal effect in improving users' perceived quality. We cannot simply treat β as the relative impact of factors because they have different units. We define the factors' relative weights as their contribution to the risk score, i.e., $\beta^t \mathbf{Z}$. When computing the contribution of a factor, the other factors are set to their respective minimum values found in the trace. The relative impact of each QoS factor, which is normalized by a total score of 100, is shown in Fig. 8. On average, the degrees of user dissatisfaction caused by the bit rate, jitter, and roundtrip time are in the proportion of 46%:53%:1%. That is, when a user hangs up because of poor or unfavorable voice quality, we believe that most of the negative feeling is caused by low bit rates (46%), and high jitter (53%), but very little is due to high round-trip times (1%).

The result indicates that *increasing the bit rate whenever appropriate would greatly enhance user satisfaction*, which is a relatively inexpensive way to improve voice quality. We do not know how Skype adjusts the bit rate as the algorithm is proprietary; however, our findings show that it is possible to improve user satisfaction by fine tuning the bit rate used. Furthermore, the fact that *higher round-trip times do not impact on users very much* could be rather a good news, as it indicates that the use of relaying does not seriously degrade user experience. Also, the fact that *jitters* have much more impact on user perception suggests that *the choice of relay node should focus more on network conditions*, i.e., the level of congestion, *rather than rely on network latency*.

4.5 User Satisfaction Index

Based on the Cox model developed, we propose the User Satisfaction Index (USI) to evaluate Skype users' satisfaction levels. As the risk score $\beta^t \mathbf{Z}$ represents the levels of instantaneous hang up probability, it can be seen as a mea-



Figure 9: Predicted vs. actual median duration of session groups sorted by their User Satisfaction Indexes.

sure of user intolerance. Accordingly, we define the User Satisfaction Index of a session as its minus risk score:

$$USI = -\beta^{t} \mathbf{Z}$$

= 2.15 × log(bit rate) - 1.55 × log(jitter)
- 0.36 × RTT,

where the bit rate, jitter, and RTTs are sampled using a two-level sampling approach as described in Section 4.4.3.

We verify the proposed index by prediction. We first group sessions by their USI, and plot the actual median duration, predicted duration, and 50% confidence bands of the latter for each group, as shown in Fig. 9. The prediction is based on the median USI for each group. Note that the y-axis is logarithmic to make the short duration groups clearer. From the graph, we observe that the logarithmic duration is approximately proportional to USI, where a higher USI corresponds to longer call duration with a consistently increasing trend. Also, the predicted duration is rather close to the actual median time, and for most groups the actual median time is within the 50% predicted confidence band.

Compared with other *objective measures* of sound quality [9,11], which often require access to voice signals at both ends, USI is particularly useful because its parameters are readily accessible; it only requires the first and second moments of the packet counting process, and the round-trip times. The former can be obtained by simply counting the number and bytes of arrived packets, while the latter are usually available in peer-to-peer applications for overlay network construction and path selection. Furthermore, as we have developed the USI based on passive measurement, rather than subjective surveys [11], it can also capture *subconscious reactions* of participants, which may not be accessible through surveys.

5. ANALYSIS OF USER INTERACTION

In this section, we validate the proposed User Satisfaction Index by an independent set of metrics that quantify the *interactivity and smoothness of a conversation*. A smooth dialogue usually comprises highly interactive and tight talk bursts. On the other hand, if sound quality is not good, or worse, the sound is intermittent rather than continuous, the level of interactivity will be lower because time is wasted waiting for a response, asking for for something to be repeated, slowing the pace, repeating sentences, and thinking. We believe that the degree of satisfaction with a call can be, at least partly, inferred from the conversation pattern.

The difficultly is that, most VoIP applications, including Skype, do not support *silence suppression*; that is, lowering the packet sending rate while the user is not talking. This design is deliberate to maintain UDP port bindings at the NAT and ensure that the background sound can be heard all the time. Thus, we cannot tell whether a user is speaking or silent by simply observing the packet rate. Furthermore, to preserve privacy, Skype encrypts every voice packet with 256-bit AES (Advanced Encryption Standard) and uses 1024 bit RSA to negotiate symmetric AES keys [2]. Therefore, parties other than call participants cannot know the content of a conversation, even if the content of voice packets have been revealed.

Given that user activity during a conversation is not directly available, we propose an algorithm that infers conversation patterns from packet header traces. In the following, we first describe and validate the proposed algorithm. We then use the voice interactivity that capture the level of user satisfaction within a conversation to validate the User Satisfaction Index. The results show that the USI, which is based on call duration, and the voice interactivity measures, which are extracted from user conversation patterns, strongly support each other.

5.1 Inferring Conversation Patterns

Through some experiments, we found that both the packet size and bit rate could indicate user activity, i.e., whether a user is talking or silent. The packet size is more reliable than the bit rate, since the latter also takes account of the packet rate, which is independent of user speech. Therefore, we rely on the packet size process to infer conversation patterns. However, deciding a packet size threshold for user speech is not trivial because the packet sizes are highly variable. In addition to voice volume, the packet size is decided by other factors, such as the encoding rate, CPU load, and network path characteristics, all of which could vary over time. Thus, we cannot determine the presence of speech bursts by simple static thresholding.

We now introduce a dynamic thresholding algorithm that is based on wavelet denoising [6]. We use wavelet denoising because packet sizes are highly variable, a factor affected by transient sounds and the status of the encoder and the network. Therefore, we need a mechanism to remove high-frequency variabilities in order to obtain a more stable description of speech. However, simple low-pass filters do not perform well because a speech burst could be very short, such as "Ah," "Uh," and "Ya," and short bursts are easily diminished by averaging. Preserving such short responses is especially important in our analysis as they indicate interaction and missing them leads to underestimation of interactivity. On the other hand, wavelet denoising can localize higher frequency components in the time domain better; therefore, it is more suitable for our scenario, since it preserves the correct interpretation of sub-second speech activity.

5.1.1 Proposed Algorithm

Our method works as follows. The input is a packet size



Figure 10: Network setup for obtaining realistic packet size processes generated by Skype

process, called the original process, which is averaged every 0.1 second. For most calls, packets are generated at a frequency of approximately 33 Hz, equivalent to about three packets per sample. We then apply the wavelet transform using the index 6 wavelet in the Daubechies family [5], which is widely used because it relatively easy to implement. The denoising operation is performed by *soft thresholding* [6] with threshold $T = \sigma \sqrt{2 \log N}$, where σ is the standard deviation of the detail signal and N denotes the number of samples. To preserve low-frequency fluctuations, which represent users' speech activity, the denoising operation only applies to time scales smaller than 1 second.

In addition to fluctuations caused by users' speech activity, the denoised process contains variations due to lowfrequency network and application dynamics. Therefore, we use a dynamic thresholding method to determine the presence of speech bursts. We first find all the local extremes, which are the local maxima or minima within a window larger than 5 samples. If the maximum difference between a local extreme and other samples within the same window is greater than 15 bytes, we call it a "peak" if it is a local maxima, and a "trough" if it is a local minima. Once the peaks and troughs have been identified, we compute the activity threshold as follows. We denote each peak or trough *i* occurring at time t_i with an average packet size s_i as (t_i, s_i) . For each pair of adjacent troughs (t_l, s_l) and (t_r, s_r) , if there are one or more peaks **P** in-between them, and the peak $p \in \mathbf{P}$ has the largest packet size, we draw an imaginary line from $(t_l, (s_l + s_p)/2)$ to $(t_r, (s_r + s_p)/2)$ as the binary threshold of user activity. Finally we determine the state of each sample as ON or OFF, i.e., whether a speech burst is present, by checking whether the averaged packet size is higher than any of the imaginary thresholds.

5.1.2 Validation by Synthesized Wave

We first validate the proposed algorithm with synthesized wave files. The waves are sampled at 22.5 KHz frequency with 8 bit levels. Each wave file lasts for 60 seconds and comprises alternate ON/OFF periods with exponential lengths of mean 2 seconds. The ON periods are composed of sine waves with the frequency uniformly distributed in the range of 500 to 2000 Hz, where the OFF periods do not contain sound.

To obtain realistic packet size processes that are contaminated by network impairment, i.e., queueing delays and packet loss, we conducted a series of experiments that involved three Skype nodes. As shown in Fig. 10, we establish a VoIP session between two local Skype hosts and force



Figure 11: Verifying the speech detection algorithm with synthesized ON/OFF sine waves



Figure 12: Verifying the speech detection algorithm with human speech recordings

them to connect via a relay node by blocking their intercommunication with a firewall. The selection of relay node is out of our control because it is chosen by Skype. The relay node used is far from our Skype hosts with transoceanic links in-between and an average RTT of approximately 350 ms, which is much longer than the average 256 ms. Also, the jitter our Skype calls experienced is 5.1 Kbps, which is approximately the 95 percentile in the collected sessions. In the experiment, we play synthesized wave files to the input of Skype on the sender, and take the packet size processes on the receiver. Because the network characteristics in our experiment are much worse than the average case in collected sessions, we believe the result of the speech detection here would be close to the worst case, as the measured packet size processes contain so much variation and unpredictability.

To demonstrate, the result of a test run is shown in Fig. 11. The upper graph depicts the original packet size process with the red line indicating true ON periods and blue checks indicating estimated ON periods. The lower graph plots the wavelet denoised version of the original process, with red and blue circles marking the location of peaks and troughs, respectively. The oblique lines formed by black crosses are binary thresholds used to determine speech activity. As shown by the figure, wavelet denoising does a good job in removing high-frequency variations that could mislead threshold decisions, but retains variations due to user speech. Note that long-term variation is present because the average packet size used in the second half of the run is significantly smaller than that in the first half. This illustrates the need for dynamic thresholding, as the packet size could be affected by many factors other than user speech.

Totally 10 test cases are generated, each of which is run 3 times. Since we have the original waves, the correctness of the speech detection algorithm can be verified. Each test is divided into 0.1-second periods, the same as the sampling interval, for correctness checks. Two metrics are defined to judge the accuracy of estimation: 1) Correctness: the ratio of matched periods, i.e., the periods whose states, either ON or OFF, are correctly estimated; and 2) the number of ON periods. As encoding, packetization, and transmission of voice data necessarily introduce some delay, the correctness is computed with time offsets ranging from minus one to plus one second, and the maximum ratio of the matched periods is used. The experiment results show that the correctness ranges from 0.73-0.92 with a mean of 0.8 and standard deviation of 0.05. The estimated number of ON periods is always close to the the actual number of ON periods; the difference is generally less than 20% of the latter. Although not perfectly exact, the validation experiment shows that the proposed algorithm estimates ON/OFF periods with good accuracy, even if the packet size processes have been contaminated by network dynamics.

5.1.3 Validation by Speech Recording

Since synthesized ON/OFF sine waves may be very different from human speech, we further experiment with human speech recordings. We repeat the preceding experiment by replacing synthesized wave files with human speech recorded via microphone during phone calls. Given that the sampled voice levels range from 0 to 255, with the center at 128, we use an offset of +/-4 to indicate whether a specific volume corresponds to audible sounds. Among a total of 9 runs for three test cases, the correctness ranges from 0.71 to 0.85. The number of ON periods differs from true values by less than 32%. The accuracy of detection is slightly worse than the experiment with synthesized waves, but still correctly captures most speech activity. Fig. 12 illustrates the result of one test, which shows that the algorithm detects speech bursts reasonably well, except for the short spikes around 47 seconds.

5.2 User Satisfaction Analysis

To determine the degree of user satisfaction from conversation patterns, we propose the following three *voice interactivity measures* to capture the interactivity and smoothness of a given conversation:

- **Responsiveness:** A smooth conversation usually involves alternate statements and responses. We measure the interactivity by the degree of "alternation," that is, the proportion of OFF periods that coincide with ON periods of the other side, as depicted in Fig. 13. Since the level of interactivity is computed separately for each direction, the minimum of both is used because it represents the worse satisfaction level.
- **Response Delay:** Short response delay, i.e., one side responds immediately after the other side stops talking, usually indicates the sound quality is good enough for the parties to understand each other. Thus, if one side starts talking when the other side is silent, we consider it as a "response" to the previous burst from the other side, and take the time difference between the two adjacent bursts as the response delay, as depicted in Fig. 13. In this way, all the response delays in the same direction can be averaged, and the larger of the average response delays in both directions is used, since longer response delay indicates poorer conversation quality.
- Talk Burst Length: This definition may not be intuitive at first glance. We believe that people usually adapt to low voice quality by *slowing their talking speed*, as this should help the other side understand. Furthermore, if people need to repeat an idea due to poor sound quality, they tend to explain the idea in another way, which is simpler, but possibly longer. Both of the above behavior patterns lead to longer speech bursts. We capture such behavior by the larger of the average burst lengths in both directions, as longer bursts indicate poorer quality. In order not to be biased by long bursts, which are due to lengthy speech or errorestimation in the speech detection, only bursts shorter than 10 seconds are considered.

Fig. 13 illustrates the voice interactivity measures proposed. Because our speech detection algorithm estimates talk bursts in units of 0.1 second, a short pause between words or sentences, either intentional or unintentional, could split one burst into two. To ensure that the estimated user activity resembles true human behavior, we treat successive bursts as a single burst if the intervals between them are shorter than 1 second in the computation of the interactivity measures. We summarize some informative statistics and voice interactivity measures of the collected sessions in Table 5.



Figure 13: Proposed measures for voice interactivity, which indicate user satisfaction from conversation patterns

 Table 5: Summary statistics for conversation patterns in the collected sessions

Statistic	Mean	Std. Dev.
ON Time	70.8%	9.5%
# ON Rate (one end)	3.9 pr/min	1.2 pr/min
# ON Rate (both ends)	6.4 pr/min	1.9 pr/min
Responsiveness	0.90	0.11
Avg. Response Time	$1.4 \mathrm{sec}$	$0.5 \sec$
Avg. Burst Length	$2.9 \sec$	$0.7 \sec$

Now that we have 1) the User Satisfaction Index (Section 4.5), which is based on the call duration compared to the network QoS model, and 2) the interactivity measures, which are inferred from the speech activity in a call. Since these two indexes are *obtained independently in completely different ways*, and the speech detection algorithm does not depend on any parameter related to call duration, we use them to cross validate their representativeness of each other. In the following, we check the correlation between the USI and the voice interactivity measures with both graphical plots and correlation coefficients.

First, we note that short sessions, i.e., shorter than 1 minute, tend to have very high indexes of responsiveness, possibly because both parties attempt to speak regardless of the sound quality during such a short conversation. Accordingly, we ignore extreme cases whose responsiveness level equals one. The scatter plot of the USI versus responsiveness is shown in Fig. 14(a). In the graph, the proportion of low-responsiveness sessions decreases as the USI increases, which supports our intuition that a higher USI indicates higher responsiveness.

Fig. 14(b) shows that response delays are also strongly related to the USI, as longer response delay corresponds to lower USI. The plot shows a threshold effect in that the average response delay does not differ significantly for USIs higher than 8. This is plausible, as response delay certainly does not decrease unboundedly, even if the voice quality is perfect. Fig. 14(c) shows that the average talk burst length consistently increases as the USI drops. As explained earlier, we consider that this behavior is due to the slow-paced conversations and longer explanations caused by poor sound quality.

We also performed statistical tests to confirm the association between the USI and the voice interactivity measures. Three measures, Pearson's product moment correlation coefficient, Kendall's τ , and Spearman's ρ , were computed, as shown in Table 6. All correlation tests reject the null hypothesis that an association does not exist at the 0.01 level. Furthermore, the three tests support each other with coefficients of the same sign and approximate magnitude.



Figure 14: The correlation between voice interactivity measures and USI.

 Table 6: Correlation tests of the USI and voice interactivity measures

	Pearson	Kendall	Spearman
Responsiveness	0.36**	0.27**	0.39**
Avg. Resp. Delay	-0.20^{**}	-0.10^{**}	-0.16^{**}
Avg. Burst Length	-0.27^{**}	-0.18^{**}	-0.26^{**}

[†] ** The p-value of the correlation test is < 0.01.

6. CONCLUSION

Understanding user satisfaction is essential for the development of QoS-sensitive applications. The proposed USI model captures the level of satisfaction without the overheads of the traditional approaches, i.e., requiring access to speech signals. It also captures factors other than signal degradation, such as talk echo, conversational delay, and subconscious reactions.

Results of the validation tests using a set of independent measures derived from user interactivities show a strong correlation between the call durations and user interactivities. This suggests that the USI based on call duration is significantly representative of Skype user satisfaction.

The best feature of the USI is that its parameters are easily accessible and computable online. Therefore, in addition to evaluating the performance of QoS-sensitive applications, the USI can be implemented as part of applications to allow adaptation for optimal user satisfaction in real time.

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